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COMBINED EFFECT OF TEMPERATURE DEPENDENT VISCOSITY AND THERMAL CONDUCTIVITY ON MHD FREE CONVECTION FLOW ALONG A VERTICAL WAVY SURFACE

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ABSTRACT
The combined effect of temperature dependent viscosity and thermal conductivity on Magnetohydrodynamic (MHD) natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated. The governing boundary layer equations are first transformed into a non-dimensional form using suitable set of dimensionless variables. The resulting nonlinear system of partial differential equations are mapped into the domain of a vertical flat plate and then solved numerically employing the implicit finite difference method, known as Keller-box scheme. The numerical results of the skin friction coefficient and the rate of heat transfer in terms of local Nusselt number, the stream lines as well as the isotherms are shown graphically for a selection of parameters set consisting of thermal conductivity parameter γ, magnetic parameter M and viscosity parameter ε for Prandtl number Pr = 7 (water) and the amplitude of the wavy surface α = 0.3

Keywords: Magnetohydrodynamic, Viscosity, Thermal Conductivity.

1. INTRODUCTION
Flow of electrically conducting fluid in the presence of magnetic field with combined effect of temperature dependent viscosity and thermal conductivity on MHD natural convection flow along a wavy surface problems are significant from the technical point of view. A considerable amount of research has been accomplished on the effects of electrically conducting fluids such as liquid metals, water mixed with a little acid and others in the presence of transverse magnetic field on the flow and heat transfer characteristics over various geometries. The viscosity and thermal conductivity of the fluid to be proportional to a linear function of temperature two semi-empirical formulae, which was proposed by Charraudeau [1]. Yao [2] first investigated the natural convection heat transfer from an isothermal vertical wavy surface and used an extended Prantdl's transposition theorem and a finite-difference scheme. Hossain et al. [3] investigated the natural convection flow past a permeable wedge with uniform surface heat flux for the fluid having temperature dependent viscosity and thermal conductivity. Parveen and Alim [4] studied effect of temperature dependent thermal conductivity on MHD natural convection flow along a vertical wavy surface. The present study is to incorporate the idea that the combined effect of temperature dependent viscosity and thermal conductivity in presence of magnetic field of electrically conducting fluid with free convection boundary layer flow along a vertical wavy surface.

2. FORMULATION OF THE PROBLEM
The boundary layer analysis outlined below allows $\bar{\sigma}(X)$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$Y_w = \bar{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right) \tag{1}$$

where L is the characteristic length associated with the wavy surface.

Under the usual Boussinesq approximation the following dimensionless form of the governing equations are obtained

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Gr^{1/4}\sigma_x \frac{\partial p}{\partial y} - Mu + \theta + \left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2} + \varepsilon\left(1+\sigma_x^2\right)\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} \tag{3}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-Gr^{1/4}\frac{\partial p}{\partial y}-\sigma_{xx}u^2 \tag{4}$$

$$\sigma_x\left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}+\varepsilon\sigma_x\left(1+\sigma_x^2\right)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\left(1+\sigma_x^2\right)(1+\gamma\theta)\frac{\partial^2\theta}{\partial y^2}+\frac{1}{\Pr}\left(1+\sigma_x^2\right)\gamma\left(\frac{\partial\theta}{\partial y}\right)^2 \tag{5}$$

In the above equations $\Pr=\frac{C_p\mu_\infty}{k_\infty}$ known is the Prandtl number, $\varepsilon=\varepsilon^*(T_w-T_\infty)$ is the dimensionless viscosity variation parameter, $\gamma=\gamma^*(T_w-T_\infty)$ is dimensionless thermal conductivity variation parameter and $M=\frac{\sigma_0\beta_0^2L^2}{\mu Gr^{1/2}}$ is the dimensionless magnetic parameter.

The boundary conditions for the present problem are

$$\left.\begin{aligned} u=v=0,\quad \theta=1 \qquad & at \quad y=0\\ u=\theta=0,\quad p=0 \qquad & as \quad y\to\infty \end{aligned}\right\} \tag{6}$$

The flow configuration of the wavy surface and the two-dimensional Cartesian coordinate system are shown in figure 1.

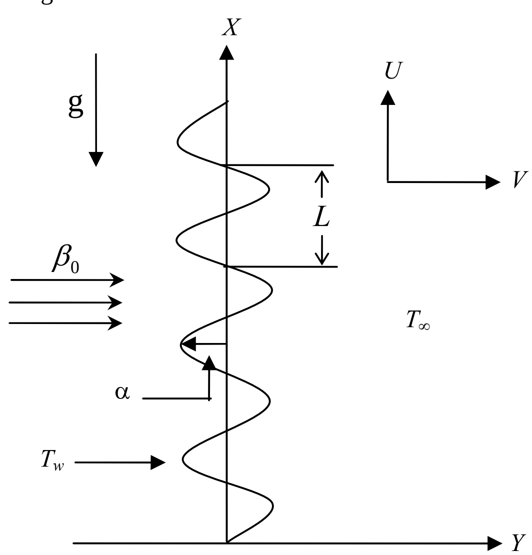


Fig 1. Physical model and coordinate system

Following Yao [2], here introduce the following non-dimensional variables

$$x=\frac{X}{L},\ y=\frac{Y-\bar{\sigma}}{L}Gr^{\frac{1}{4}},\ p=\frac{L^2}{\rho\nu^2}Gr^{-1}P$$

$$u=\frac{\rho L}{\mu_\infty}Gr^{-1/2}U,\quad v=\frac{\rho L}{\mu_\infty}Gr^{-1/4}\left(V-\sigma_x U\right) \tag{7}$$

$$\sigma_x=\frac{d\bar{\sigma}}{dX}=\frac{d\sigma}{dx},\ Gr=\frac{g\beta(T_w-T_\infty)}{\nu^2}L^3,\ \theta=\frac{T-T_\infty}{T_w-T_\infty}$$

It can easily be seen that the convection induced by the wavy surface is described by equations (2)–(5). We further notice that, equation (4) indicates that the pressure gradient along the y-direction is $O(Gr^{-1/4})$, which implies that lowest order pressure gradient along x-direction can be determined from the inviscid flow solution. For the present problem this pressure gradient ($\partial p/\partial x=0$) is zero. Because the pressure along x-direction turns into convective motion of fluid. Equation (4) further shows that $Gr^{1/4}\partial p/\partial y$ is $O(1)$ and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from equations (3) and (4) leads to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2+\varepsilon\left(1+\sigma_x^2\right)\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial y}-\frac{M}{1+\sigma_x^2}u+\frac{1}{1+\sigma_x^2}\theta \tag{8}$$

The variable viscosity and thermal conductivity chosen in this study that is introduced by Charraudeau [1] and used by Hossain et al. [3] as follows:

$$\mu=\mu_\infty[1+\varepsilon^*(T-T_\infty)] \tag{9}$$

$$k=k_\infty\left[1+\gamma^*\left(T-T_\infty\right)\right] \tag{10}$$

where μ_∞ is the viscosity and k_∞ is the thermal conductivity of the ambient fluid.

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi=x^{3/4}f(x,\eta),\quad \eta=yx^{-1/4},\quad \theta=\theta(x,\eta) \tag{11}$$

where $f(\eta)$ is the dimensionless stream function, η is the pseudo similarity variable and ψ is the stream function that satisfies the continuity equation (2) and is related to the velocity components in the usual way as

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \tag{12}$$

Introducing the transformations given in equation (11) and using (12) into equations (8) and (5) are transferred to the new co-ordinate system. Thus the resulting equations are

$$(1+\sigma_x^2)(1+\varepsilon\theta)f'''+\frac{3}{4}ff''-\left(\frac{1}{2}+\frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2+\frac{1}{1+\sigma_x^2}\theta+\varepsilon(1+\sigma_x^2)\theta' f''-\frac{Mx^{1/2}}{1+\sigma_x^2}f'=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \tag{13}$$

$$\frac{1}{\Pr}\left(1+\sigma_x^2\right)\left(1+\gamma\theta\right)\theta''+\frac{1}{\Pr}\left(1+\sigma_x^2\right)\gamma\theta'^2 + \frac{3}{4}f\theta' = x\left(f'\frac{\partial\theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right) \qquad (14)$$

The boundary conditions as mentioned in equation (6) then take the form given below:

$$\left.\begin{array}{ll} f(x,o)=f'(x,o)=0, & \theta(x,o)=1 \\ f'(x,\infty)=0, & \theta(x,\infty)=0 \end{array}\right\} \qquad (15)$$

In the above equations prime denote the differentiation with respect to η.

The rate of heat transfer in terms of the local Nusselt number Nu_x and the local skin friction coefficient C_{fx} take the following forms:

$$Nu_x (Gr/x)^{-1/4} = -(1+\gamma)\sqrt{1+\sigma_x^2}\,\theta'(x,o) \qquad (16)$$

$$C_{fx} (Gr/x)^{1/4}/2 = (1+\varepsilon)\sqrt{1+\sigma_x^2}\, f''(x,o) \qquad (17)$$

3. METHOD OF SOLUTION

The free convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface in presence of magnetic field with variable viscosity and thermal conductivity has been investigated. The governing equations (13) and (14) with the boundary conditions (15) are solved numerically using the very efficient implicit finite difference method known as Keller box scheme developed by Keller [5].

4. RESULTS AND DISCUSSION

Numerical values of the skin friction coefficient C_{fx}, the rate of heat transfer in terms of the Nusselt number Nu_x, the streamlines and the isotherms are obtained for different values of the viscosity parameter ε = 0.0 (constant viscosity) to 30.0, the magnetic parameter M = 0.0 (non magnetic field) to 5.0 and thermal conductivity variation parameter γ ranging from 0.0 (constant thermal conductivity) to 15.0 and depicted in figures 2-10. Figures 2(a) and 2(b) are depicted graphically the influence of ε on the surface shear stress in terms of the local skin friction coefficient coefficient C_{fx} and the rate of heat transfer Nu_x respectively keeping all other controlling parameters amplitude of wavy surface α = 0.3, M = 1.0, thermal conductivity variation parameter γ = 4.0 and Prandtl number Pr = 7.0. Figure 2(a) indicates that increasing the values of the viscosity variation parameter ε the skin friction coefficient increases monotonically along the upward direction of the plate and it is seen that the local skin friction coefficient C_{fx} increases by 66.17% as ε changes from 0.0 to 30.0. The rate of heat transfer decreases by 46.08% due to the increased value of ε can be shown from figure 2(b).

The variation of thermal conductivity variation parameter γ = (0.0, 2.0, 6.0, 10.0, 15.0) on the skin friction coefficient and the heat transfer coefficient while Prandtl number Pr = 7.0, the amplitude of the wavy surface α = 0.3, viscosity variation parameter ε = 5.0 and magnetic parameter M = 0.8 are shown in figures 3(a)-3(b) respectively. The increasing value of γ the skin friction coefficient and heat transfer coefficient increases monotonically along the upward direction of the plate. It is observed that the skin friction coefficient and heat transfer rate increases by 40.09% and 80.75% respectively when γ changes from 0 to 15.0.

The effects of the magnetic parameter M the local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x are illustrated in figures 4(a) and 4(b) respectively for Prandtl number Pr = 7.0, amplitude of wavy surface α = 0.3, thermal conductivity variation parameter γ = 5.0 and viscosity parameter ε = 5.0. The skin friction coefficient and the rate of heat transfer coefficient decreases by 34.39% and 12.26% respectively as M increases from 0.0 to 5.0.

The effect of the temperature dependent viscosity variation parameter ε on the development of streamlines and isotherms are plotted in figures 5 and 6 respectively for Prandtl number Pr = 7.0, amplitude of wavy surface α = 0.3, thermal conductivity variation parameter γ = 4.0 and magnetic parameter M = 1.0. It is found that for ε = 0.0 the value of ψ_{max} is 2.86, for ε = 10.0 ψ_{max} is 2.49, for ε = 20.0 ψ_{max} is 2.29 and for ε = 30.0 ψ_{max} is 2.10. Hence from these figures it is seen that the effect of ε, the flow rate in the boundary layer decreases and the thermal boundary layer thickness increases monotonically.

Figures 7 and 8 show the effect of thermal conductivity variation parameter γ on the formation of streamlines and isotherms respectively with other controlling parameters amplitude of wavy surface α = 0.3, magnetic parameter M = 0.8, viscosity parameter ε = 5.0 and Prandtl number Pr = 7.0. It can be noted that for γ equal to 0.0, 6.0, 10.0, and 15.0 the maximum values of ψ, that is, ψ_{max} are 1.62, 3.26, 4.06 and 4.58 respectively. So it can be concluded that for large value of γ both the momentum and the thermal boundary layer thickness increases.

The effect of different values of magnetic parameter M equal to 0.0, 0.5, 3.0 and 5.0 on the streamlines and isotherms are illustrated in figures 9 and 10 respectively with other controlling parameters Prandtl number Pr = 7.0, amplitude of wavy surface α = 0.3, γ = 5.0 and ε = 5.0. Figure 9 depicts that the maximum values of ψ decreases quickly while the values of M increases. When M = 0.0 the value of ψ_{max} is 5.09, for M = 0.5 the value of ψ_{max} is 3.35, for M = 3.0 the value of ψ_{max} is 1.83 and for M = 5.0 the value of ψ_{max} is 1.35. On the other hand temperature distribution increases significantly as the values of magnetic parameter M increases which presented in figure 10.

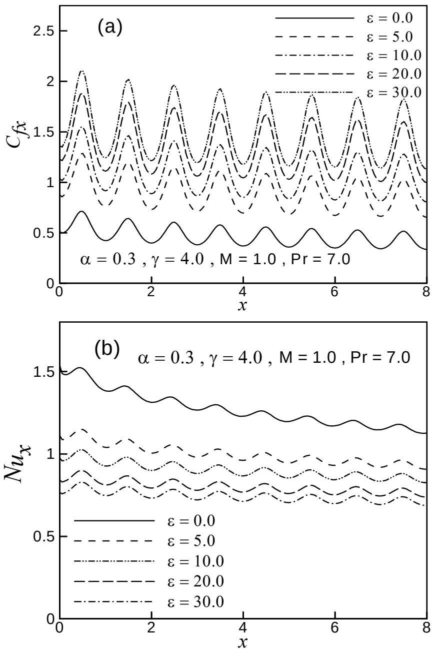


Fig 2. Variation of (a) skin-friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for varying of ε while $\alpha = 0.3$, $M = 1.0$, $\gamma = 4.0$ and Pr = 7.0.

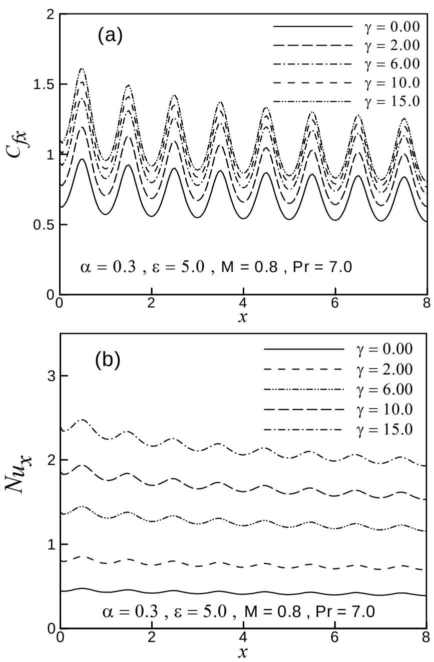


Fig 3. Variation of (a) skin-friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for varying of γ while Pr = 7.0, $M = 0.8$, $\varepsilon = 5.0$ and $\alpha = 0.3$.

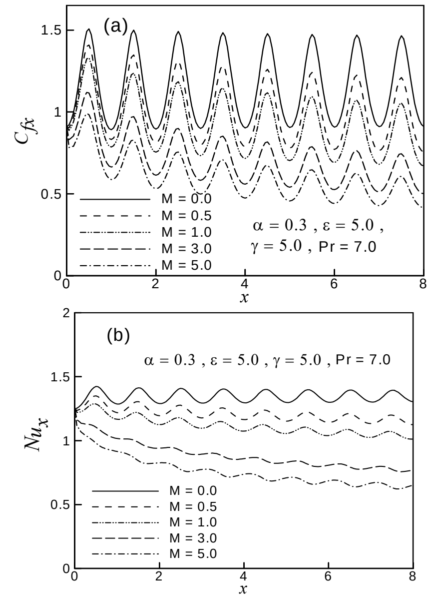


Fig 4. Variation of (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for different values of magnetic parameter M while Pr = 7.0, $\alpha = 0.3$, $\gamma = 5.0$ and $\varepsilon = 5.0$.

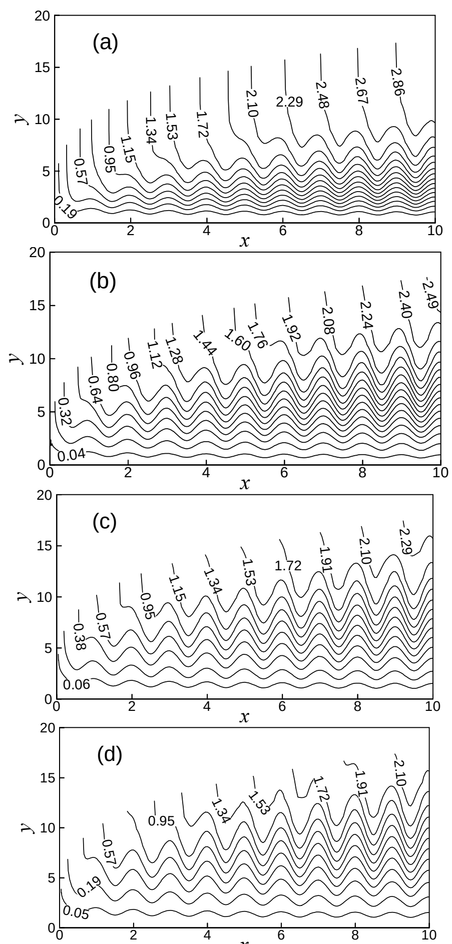


Fig 5. Streamlines for (a) $\varepsilon = 0.0$ (b) $\varepsilon = 10.0$ (c) $\varepsilon = 20.0$ (d) $\varepsilon = 30.0$ while Pr = 7.0, $M = 1.0$, $\gamma = 4.0$ and $\alpha = 0.3$.

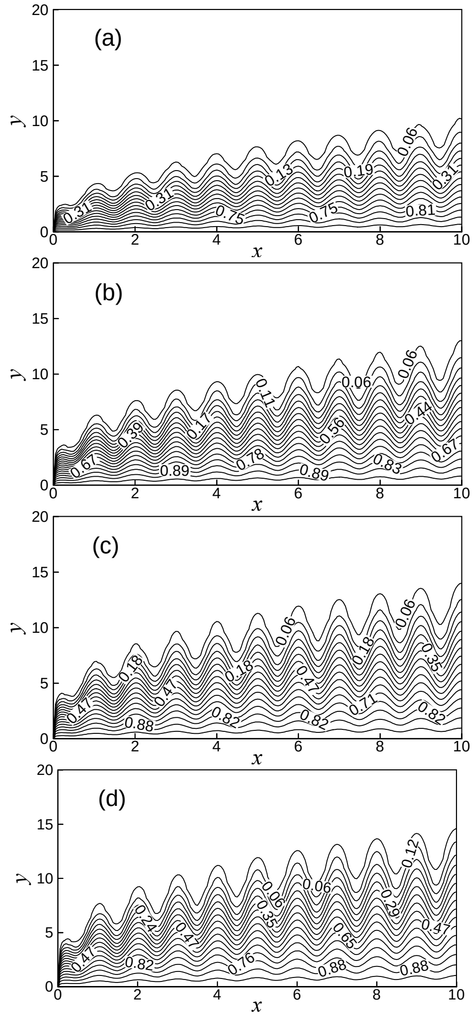
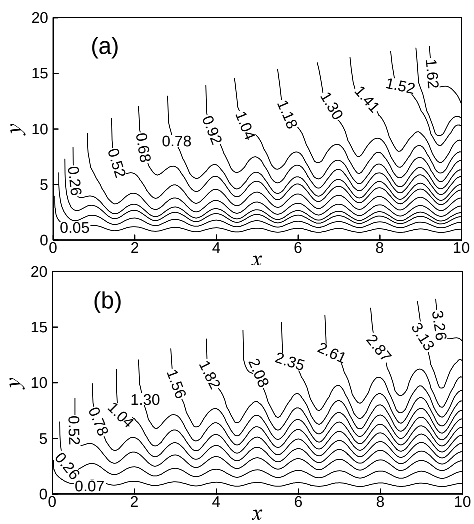


Fig 6. Isotherms for (a) $\varepsilon = 0.0$ (b) $\varepsilon = 10.0$ (c) $\varepsilon = 20.0$ (d) $\varepsilon = 30.0$ while Pr = 7.0, $M = 1.0$, $\gamma = 4.0$ and $\alpha = 0.3$.



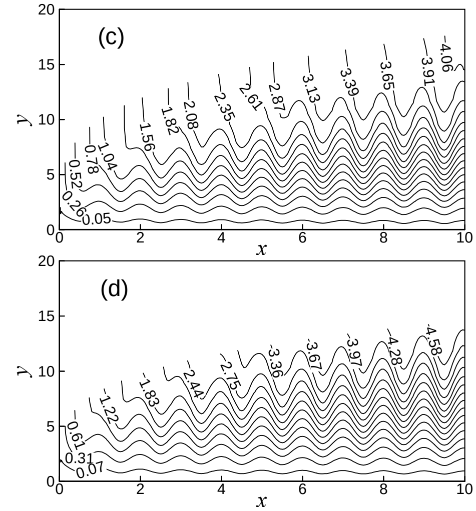


Fig 7. Streamlines for (a) $\gamma = 0.0$ (b) $\gamma = 6.0$ (c) $\gamma = 10.0$ (d) $\gamma = 15.0$ while Pr = 7.0, $M = 0.8$, $\alpha = 0.3$ and $\varepsilon = 5.0$.

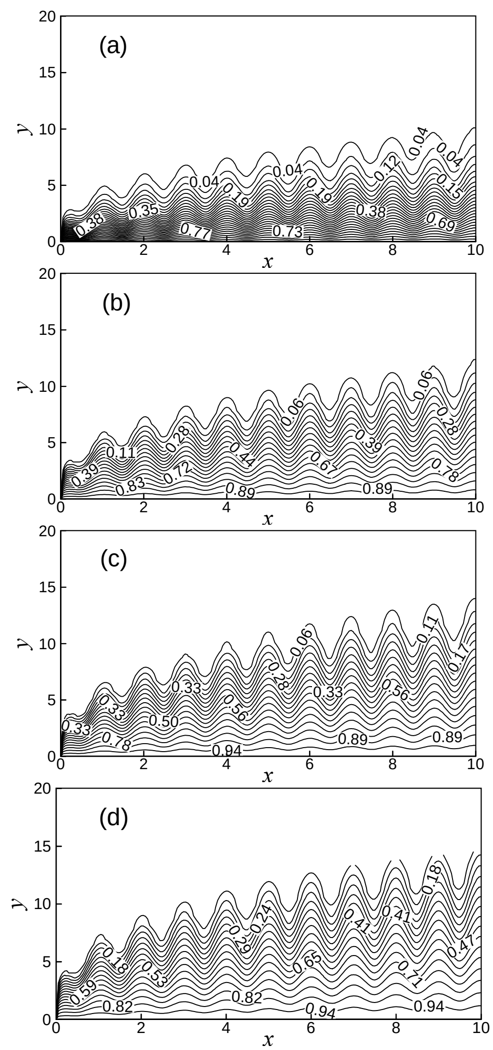


Fig 8. Isotherms for (a) $\gamma = 0.0$ (b) $\gamma = 6.0$ (c) $\gamma = 10.0$ (d) $\gamma = 15.0$ while Pr = 7.0, $M = 0.8$, $\alpha = 0.3$ and $\varepsilon = 5.0$.

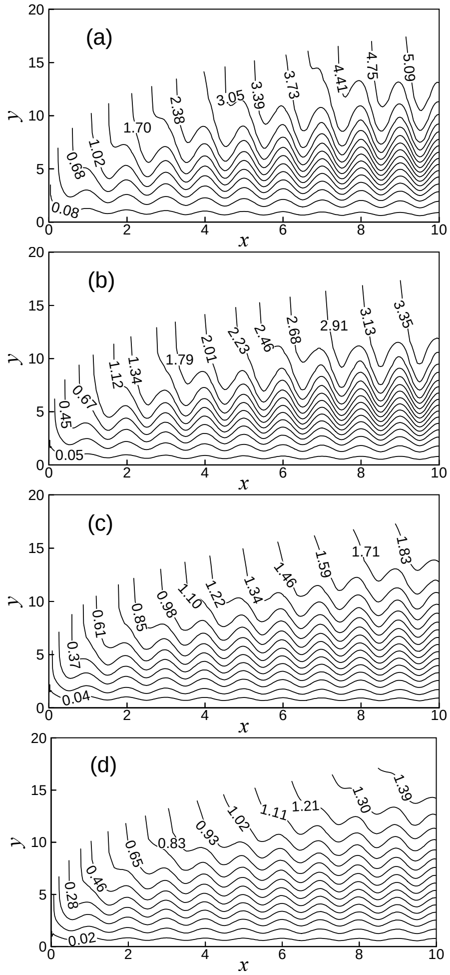
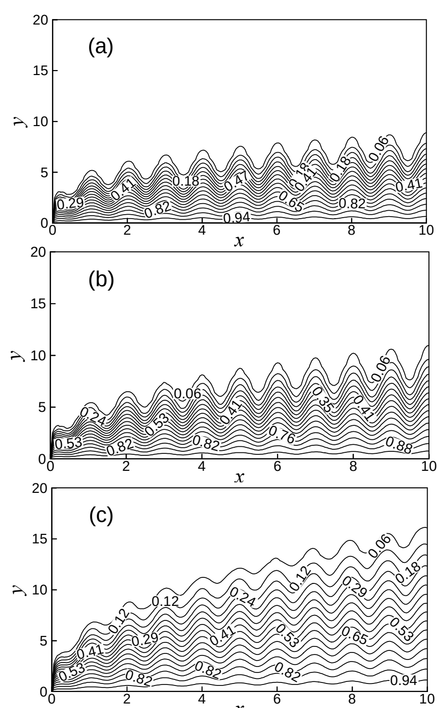


Fig 9. Streamlines for (a) M = 0.0 (b) M = 0.5 (c) M = 3.0 (d) M = 5.0 while Pr = 7.0, γ = 5.0, ε = 5.0 and α = 0.3.



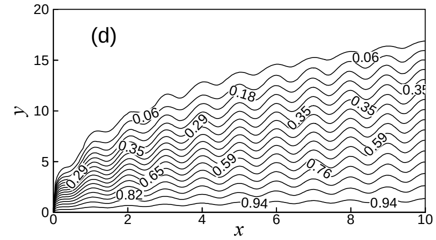


Fig 10. Isotherms for (a) M = 0.0 (b) M = 0.5 (c) M = 3.0 (d) M = 5.0 while Pr = 7.0, γ = 5.0, ε = 5.0 and α = 0.3.

5. CONCLUSIONS

The combined effect of temperature dependent viscosity and thermal conductivity on MHD free convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated. The conclusions are as follows:

- The effect of increasing viscosity parameter ε results in increasing the local skin friction coefficient C_{fx}, the thermal boundary layer thickness and decreasing the local rate of heat transfer Nu_x and the velocity boundary layer thickness over the whole boundary layer.
- An increase in the values of M leads to decrease the skin friction coefficient C_{fx}, the local rates of heat transfer Nu_x and the velocity profile while the reverse phenomena occurs for the temperature distribution.
- It is found that the skin friction coefficient, heat transfer rates, the flow rate and thermal boundary layer thickness increases within the boundary layer for the increasing value of thermal conductivity variation parameter γ.

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